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Coloured noise influence on system evolution

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Abstract. Within the power–law approach for noise amplitude dependence on stochastic variables, we present a picture of noise–induced transitions in systems affected by coloured multiplicative noise. The governed equations for main statistical moments are obtained and investigated in detail. We show that a reentrant noise–induced transition is realized within a window of the control parameter.

PACS. 05.40.-a Fluctuation phenomena, random processes, noise and Brownian motion – 47.20.Ky Nonlinearity (including bifurcation theory)

1 Introduction

The study of dynamical systems perturbed by noise is recurrent in many contexts of physics and other sciences [1]. Noise is known to play crucial role in systems out–of–equilibrium where the variation of the noise intensity leads to dramatic changes [2]. In fact, zero-dimensional systems can undergo noise–induced unimodal–bimodal transitions, which are not reduced to phase transitions in usual thermodynamical sense. Such type of transitions are described in terms of the most probable value of the stochastic variable, which reflects the appearing new maxima of the probability density function [3]. As a rule, noise–induced transitions keep symmetry of distribution functions due to the fact that the first statistical moment equals zero.

In contrary, phase transitions in d–dimensional systems, for which the symmetry breaking is inherent, are described in terms of the first statistical moment that plays a role of the order parameter to measure the above asymmetry. Making use of the mean–field approach in d–dimensional systems shows the crucial role of inhomogeneity that may change the picture of the transition drastically (reentrant order–disorder phase transitions) [4–6]. On the other hand, a self–consistent evolution of stochastic systems affected by white noise with growing intensity may even provide symmetry breaking in zero–dimensional systems [7]. It allows one to represent the noise–induced transition along the lines of usual thermodynamical approach [8] when the first statistical moment $\langle x\rangle$ is added by variance $\langle(\delta x)^2\rangle$.

In this work, we start with a stochastic differential equation with a force defined through x^4–potential and a coloured multiplicative noise. It allows us to obtain the governed equations for the first statistical moment and variance to describe a self–consistent picture of the noise–induced transitions in usual manner. In this line, we associate the state $\langle x\rangle \neq 0$ as an ordered phase and state $\langle x\rangle = 0$ as a disordered one. Giving integral characteristics of the probability density function, such representation allows us to get a deep insight to the picture of the noise–induced transition following from explicit form of this function [9].

The paper is organized in the following manner. In Section 2 we present the main assumptions and the basic equations of our approach. Sections 3 and 4 are devoted to considering the evolution of the disordered system and the system with the ordered state, respectively. We show that $\langle x\rangle$ behaves itself in a nontrivial manner in a window of noise correlation time and control parameter. Finally, Section 5 contains concluding remarks.

2 Model and basic equations

In the simplest form, the problem of coloured noise can be introduced by considering a relevant macrovariable $x(t)$ (density of a given physical quantity) that satisfies a stochastic differential equation of the form

$$\dot{x} = f_0(x) + g_0(x)\zeta(t), \tag{1}$$

where $f_0(x)$ represents a deterministic force. The influence of the bath is represented through the second term being a fluctuating force with an amplitude $g_0(x)$. Here $\zeta(t)$ is the random term, quite often assumed to be gaussianly distributed. Without loss of generality, we take the deterministic force in the form

$$f_0(x) = \varepsilon x - x^3, \tag{2}$$

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which can be obtained from a bistable potential, $f_0(x) = -V'(x)$, with

$$V(x) = -\frac{\varepsilon}{2}x^2 + \frac{1}{4}x^4. \tag{3}$$

Here ε is the parameter that acts as a dimensionless temperature, counted from the critical value.

Considering a whole set of models with typical behaviour we can use the power–law approximation for the noise amplitude

$$g_0(x) = x^a, \qquad a \in [0,1]. \tag{4}$$

Such a constraint allows us to describe a large family of models which belong to the systems with a self–similar phase space. In the particular cases we can pass to the ordinary thermodynamic system ($a = 0$), directed percolation model ($a = 1/2$), population dynamics and forest fires ($a = 1$).

In the simplest case for $\zeta(t)$ we can use the definition of the Ornstein–Uhlenbeck process

$$\tau\dot{\zeta} = -\zeta + \xi(t), \tag{5}$$

where τ is the correlation time, $\xi(t)$ is the white noise source ($\langle\xi(t)\rangle = 0$, $\langle\xi(t)\xi(t')\rangle = \delta(t-t')$).

If we take the time derivative of equation (1) and replace first $\dot{\zeta}$ in terms of ζ and ξ from equation (5) and then ζ in terms of $\dot{x}$ and x from equation (1) we can obtain the non–Markovian stochastic differential equation

$$\tau\left(\ddot{x} - \frac{g_0'}{g_0}\dot{x}^2\right) + \sigma\dot{x} = f_0 + g_0\xi(t), \tag{6}$$

where

$$\sigma = \left[1 - \tau\left(f_0' - f_0\frac{g_0'}{g_0}\right)\right]. \tag{7}$$

According to the unified coloured noise approximation [6,10] we use the adiabatic elimination (neglecting $\ddot{x}$) and neglect $\dot{x}^2$. Moreover, we have to note that the $\xi(t)$-term in Equation (5) is the white noise and (obeying the mathematical rules) we pass to the Itô interpretation (we account for the physics of the process with the help of Eq. (5)).

The problem now lies in obtaining an evolution equation for the order parameter η. Averaging the reduced stochastic equation, we get the equation of motion in the form

$$\langle\sigma(x)\dot{x}\rangle = \langle f_0(x)\rangle. \tag{8}$$

The term on the left hand side can be represented as a full derivative, *i.e.* $\mathrm{d}y(x)/\mathrm{d}t = \sigma\mathrm{d}x/\mathrm{d}t$, and after averaging, following [1], we get $\langle\mathrm{d}y(x)/\mathrm{d}t\rangle = \langle\sigma\mathrm{d}x\rangle/\mathrm{d}t$. Introducing the notation for the autocorrelator $S = \langle(\delta x)^2\rangle$ the left hand side of equation (8) reads

$$\langle\sigma(x)\dot{x}\rangle = \dot{\eta}(\epsilon + \kappa(\eta^2 + S)) + \kappa\eta\dot{S}, \tag{9}$$

where

$$\epsilon = 1 - \varepsilon\tau(1-a), \qquad \kappa = \tau(3-a). \tag{10}$$

The resulting equation for the first statistical moment now reads

$$[\epsilon + \kappa(\eta^2 + S)]\dot{\eta} + \kappa\eta\dot{S} = \eta(\varepsilon - \eta^2) - 3\eta S. \tag{11}$$

Now our goal is to construct an evolution equation for the autocorrelator. We exploit the conventional device and proceed from the following differential: $\mathrm{d}y^2 = 2y\mathrm{d}y + (\mathrm{d}y)^2$. According to the aforementioned stochastic process $\mathrm{d}y(x(t))$ we find: $\mathrm{d}y^2 = \epsilon^2\mathrm{d}x^2 + (2\epsilon\kappa/3)\mathrm{d}x^4 + (\kappa/3)^2\mathrm{d}x^6$. Making use of the supposition $x^6 \ll 1$, the reduced equation for the second moment can be rewritten as $\langle\mathrm{d}y^2/\mathrm{d}t\rangle \approx \epsilon^2\mathrm{d}\langle x^2\rangle/\mathrm{d}t + (2\epsilon\kappa/3)\mathrm{d}\langle x^4\rangle/\mathrm{d}t$. In the common mathematical notation for the stochastic differential we have

$$\mathrm{d}y = f_0(x)\mathrm{d}t + g_0(x)\mathrm{d}W, \tag{12}$$

where $\mathrm{d}y$ is the increment of the solution process and $\mathrm{d}W$ is the increment of the Wiener process. Moreover, $\mathrm{d}W$ is the statistically independent of the random variable $y(t')$ for $t' \le t$, and $\mathrm{d}W^2$ can be replaced by $\mathrm{d}t$. This enables one to write down the equation for the variance in the form

$$2\epsilon\eta\left[\epsilon + 4\kappa\left(\frac{1}{3}\eta^2 + S\right)\right]\dot{\eta} + \epsilon[\epsilon + 4\kappa(\eta^2 + S)]\dot{S} = 2\left[\epsilon\varepsilon(\eta^2 + S) - \left(\epsilon - \frac{\kappa\varepsilon}{3}\right)(\eta^4 + 6\eta^2 S + 3S^2)\right] + \langle x^{2a}\rangle. \tag{13}$$

This equation combines the integer order averages and the fractional one, namely $\langle g_0^2(x)\rangle = \langle x^{2a}\rangle$. Since it is not possible to find an analytical closed form for the corresponding moment $\langle x^{2a}\rangle$ as a function of $\langle x\rangle$ or/and $\langle x^2\rangle$. It is usually replaced by a function with the same asymptotic properties which, ultimately, determine the global features of the process. A possible form of such an approximation was given in [7]. Here we use the same suppositions. We assume that the distribution function of the system states is a homogeneous function, *i.e.*

$$P(x) \approx Ax^{-2a}, \qquad A \equiv \frac{1}{2}|1-2a|b^{|1-2a|}, \tag{14}$$

where the cut-off parameter $b \to 0$. It provides the following approximation:

$$\langle x^{nq}\rangle = \alpha_n(q)\langle x^n\rangle^{p_n(q)} \tag{15}$$

where

$$p_n(q) = \frac{1-2a+nq}{1-2a+n},$$
$$\alpha_n(q) = A^{\frac{n(1-q)}{1-2a+n}}p_n^{-1}(q)(1-2a+n)^{p_n(q)-1}, \tag{16}$$

n is an integer number, q takes a fractional magnitude. A keypoint of the system with the multiplicative noise (4) is that its behaviour is governed by the noise exponent a.

At $1/2 < a < 1$, when the fractal dimension of the phase space $D = 2(1-a)$ is less than 1, the system is always disordered and its evolution is represented by the autocorrelator $S(t)$. It provides $q > 1$ in equation (15), hence the fractional order average is expressed through the autocorrelator $S(t)$. In the case $a < 1/2$, where $D > 1$, the system undergoes noise–induced phase transition and in the fractional order average we account a contribution given by the order parameter η (here $q < 1$ in Eq. (15))[1]. Therefore, we can rewrite equation (15) in the form

$$\langle x^{2a}\rangle = \begin{cases} \alpha_1 \eta^{p_1}, & 0 < a < \frac{1}{2}, \\ \alpha_2 S^{p_2}, & \frac{1}{2} < a < 1, \end{cases} \tag{17}$$

where the multiplier and exponent are defined as

$$\alpha_1 = A^{(1-2a)p_1} p_1^{-p_1}, \quad p_1 = \frac{1}{2(1-a)},$$
$$\alpha_2 = A^{2(1-a)p_2} p_2^{-p_2}, \quad p_2 = \frac{1}{(3-2a)}. \tag{18}$$

3 Evolution of disordered system

Let us consider the evolution of the system in the domain $a > 1/2$, at first. In such a case the system is affected by the absorbing state $x = 0$. The singularity of the probability density function does not allow passage to other states except absorbing one. Therefore, for the first statistical moment we have $\eta(t) = 0$. Hence, the system can be considered as the disordered one. Therefore, the evolution of the system is governed by the equation for the autocorrelator

$$\dot{S}\left(\frac{\epsilon}{2} + 2\kappa S\right) = S\left(\varepsilon - S\left(3 - \frac{\varepsilon\kappa}{\epsilon}\right)\right) + \alpha_2 S^{p_2}. \tag{19}$$

Time dependencies of the autocorellator are presented in Figure 1. It is seen that $S(t)$ monotonically attains the stationary magnitude determined by the equation

$$\varepsilon - \left(3 - \frac{\varepsilon\kappa}{\epsilon}\right) S_0 + \alpha_2 S_0^{p_2 - 1} = 0 \tag{20}$$

at condition $S_0 \neq -\epsilon/4\kappa$. In Figure 2 we plot steady states at different values of the noise correlation time τ and different values of the control parameter ε. According to equation (20), with an ε or τ increase the stationary value S_0 rises from the minimal magnitude. Let us pass to the limit $S \ll 1$. If we put $S^{p_2} \gg S \gg S^2$ then equation (19) gives the power–law time dependence

$$S_{t\to 0} = B t^{\frac{1}{1-p_2}}, \quad B = \left(\frac{2(1-p_2)\alpha_2}{\epsilon}\right)^{\frac{1}{1-p_2}}. \tag{21}$$

In the opposite case $S_0 - S \ll S_0$ one has an exponential form $S - S_0 \propto e^{\lambda t}$, where $\lambda = 2\epsilon^{-1}[\varepsilon(1-p_2) + S_0 p_2(3 - \varepsilon\kappa/\epsilon)]$.

[1] Here we need to stress that unimodal–bimodal noise–induced transitions, where x_0 is the order parameter, can be realized at arbitrary magnitude of the exponent a.

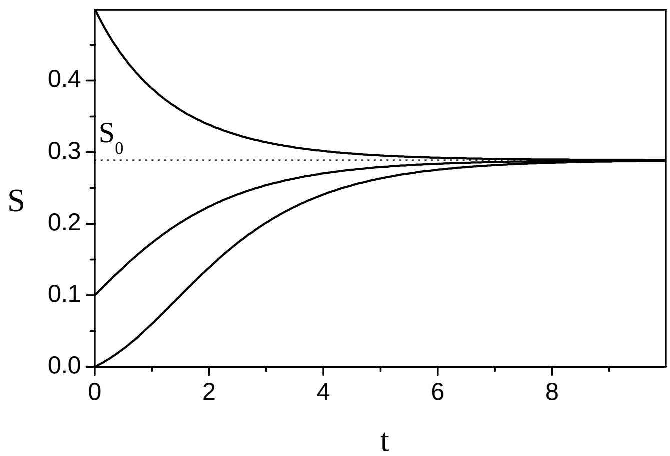


Fig. 1. Time dependence of the autocorrelator S at $\varepsilon = 0.6$, $\tau = 0.5$, $a = 0.8$.

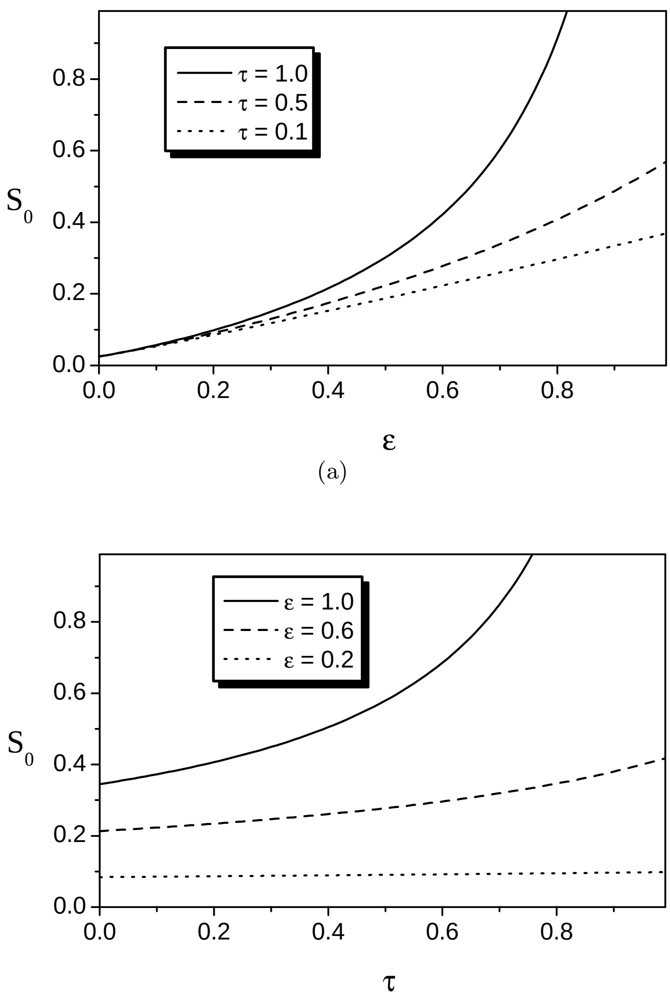


Fig. 2. Stationary states of the system at $a = 0.8$: (a) S_0 *vs.* control parameter at different values of τ; (b) S_0 *vs.* noise correlation time at different values of ε.

4 Evolution of ordering system

A more interesting situation of the system behaviour can be observed in the case $a < 1/2$. At small values of the exponent a the character of the boundary $x = 0$ is changed and the absorbing state disappears. In such a case we have a nontrivial magnitude for the first moment $\langle x(t)\rangle$. Therefore, the dynamics of the system is governed by equations for the order parameter and autocorrelator

$$\gamma(\eta,S)\dot{\eta} = \eta[\epsilon - \eta^2 - 3S][\epsilon + 4\kappa(\eta^2+S)] - 2\kappa\eta\left[\varepsilon(\eta^2+S) - \left(1-\frac{\kappa\varepsilon}{3\epsilon}\right)(\eta^4+6\eta^2S+3S^2)\right] - \kappa\epsilon\alpha_1\eta^{p_1+1}, \quad (22)$$

$$\beta(\eta,S)\dot{S} = \left[\varepsilon(\eta^2+S) - \left(1-\frac{\kappa\varepsilon}{3\epsilon}\right)(\eta^4+6\eta^2S+3S^2)\right] \times [\epsilon+\kappa(\eta^2+S)] - \eta^2\left[\epsilon+4\kappa\left(\frac{\eta^2}{3}+S\right)\right][\varepsilon-\eta^2-3S] + [\epsilon+\kappa(\eta^2+S)]\alpha_1\eta^{p_1}, \quad (23)$$

where we use the following notations

$$\gamma(\eta,S) = [\epsilon+\kappa(\eta^2+S)][\epsilon+4\kappa(\eta^2+S)] - 2\eta^2\kappa\left[\epsilon+4\kappa\left(\frac{\eta^2}{3}+S\right)\right], \quad (24)$$

$$\beta(\eta,S) = \left[\frac{\epsilon}{2}+2\kappa(\eta^2+S)\right][\epsilon+\kappa(\eta^2+S)] - \kappa\eta^2\left[\epsilon+4\kappa\left(\frac{\eta^2}{3}+S\right)\right]\cdot \quad (25)$$

The obtained closed-loop system of differential equations can be analyzed with the help of the phase plane method. From the corresponding phase portrait (Fig. 3a) it is seen that at small magnitudes of the control parameter ε there is only one attractive point C_0 with coordinates $\eta_0 = 0$, $S_0 = (\varepsilon/3)[(1-\varepsilon\tau(1-a))/(1-2\varepsilon\tau(1-2a/3))]$.

If the control parameter increases then saddle and attractive points are appeared. Coordinates of these points are given as solutions of the stationary equations

$$[\epsilon-\eta_0^2-3S_0][\epsilon+4\kappa(\eta_0^2+S_0)] = 2\kappa\left[\varepsilon(\eta_0^2+S_0)-\left(1-\frac{\kappa\varepsilon}{3\epsilon}\right)(\eta_0^4+6\eta_0^2S+3S_0^2)\right] + \kappa\epsilon\alpha_1\eta_0^{p_1}, \quad (26)$$

$$\left[\varepsilon(\eta_0^2+S_0)-\left(1-\frac{\kappa\varepsilon}{3\epsilon}\right)(\eta_0^4+6\eta_0^2S_0+3S_0^2)\right] \times [\epsilon+\kappa(\eta_0^2+S_0)] = \eta_0^2\left[\epsilon+4\kappa\left(\frac{\eta_0^2}{3}+S_0\right)\right][\varepsilon-\eta_0^2-3S_0] - [\epsilon+\kappa(\eta_0^2+S_0)]\alpha_1\eta_0^{p_1}. \quad (27)$$

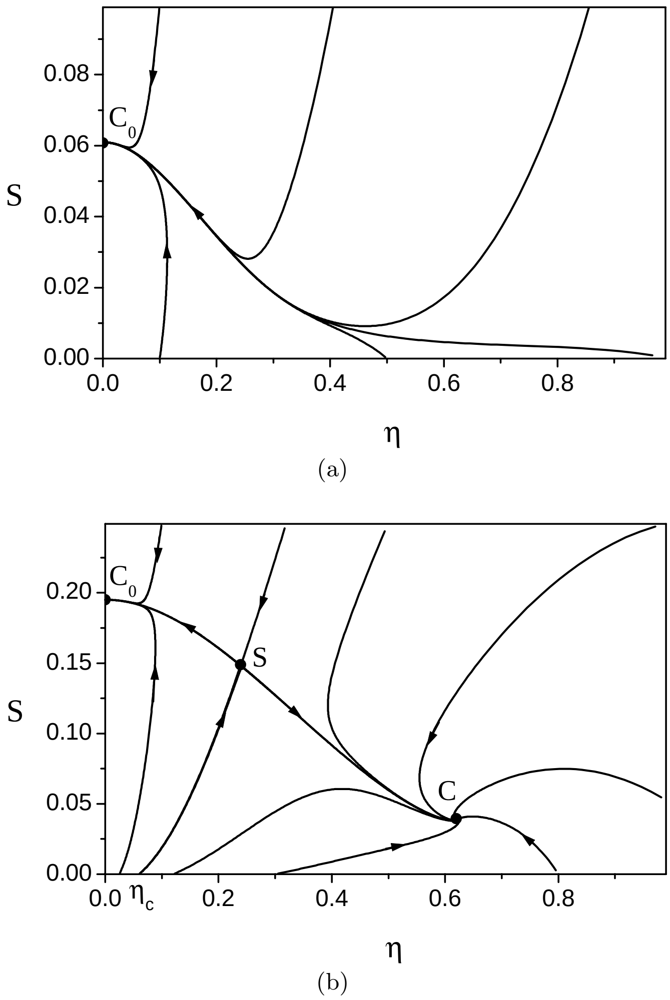


Fig. 3. Phase portraits at $a = 0.2$: (a) $\varepsilon = 0.1$, $\tau = 0.01$; (b) at $\varepsilon = 0.5$, $\tau = 0.2$.

The phase diagram in Figure 4 illustrates the appearance of the domain of ordered state at small values of the noise correlation time. If we increase the noise intensity (given by the exponent a) the noise–induced phase is realized at large magnitudes of the control parameter. Moreover, for a range of values of τ the ordered state exists within a window of the control parameter. It means that the noise correlations of the weak multiplicative noise creates the ordered state at $\varepsilon_0^{(1)}$ and destroys it at $\varepsilon > \varepsilon_0^{(1)}$. The domain of the noise–induced state is decreased with the noise correlation time growth. Hence, if τ large enough then the system becomes disordered. Therefore, the ordered state appearing is possible if the multiplicative noise has a short memory. Dependencies of the steady states are shown in Figures 5 and 6. Here thin lines display the saddle point S and thick lines correspond to the attractive point C in Figure 3b. Some distinctive feature can be seen from Figures 5 and 6: the system undergoes a transition of the first order despite the fact that the bare x^4-potential corresponds to the continuous one. So, the noise can change the kind of the transition.

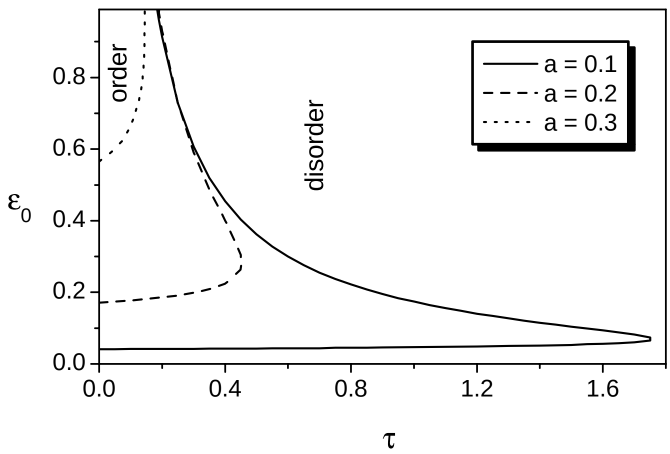
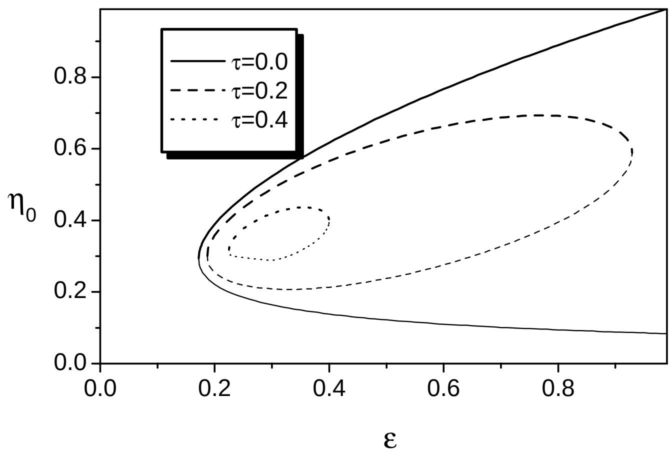
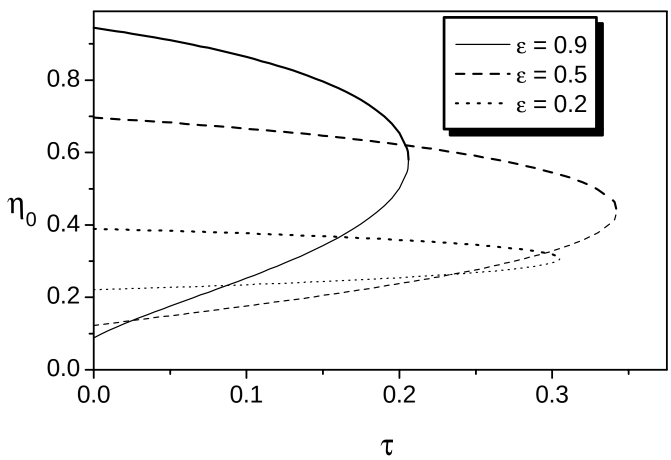


Fig. 4. Phase diagram.

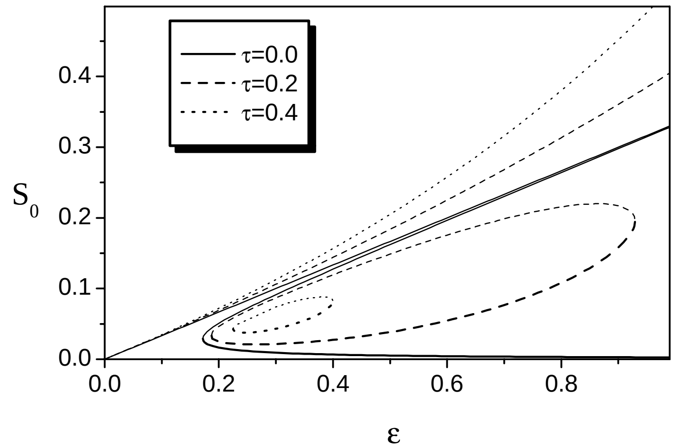


(a)

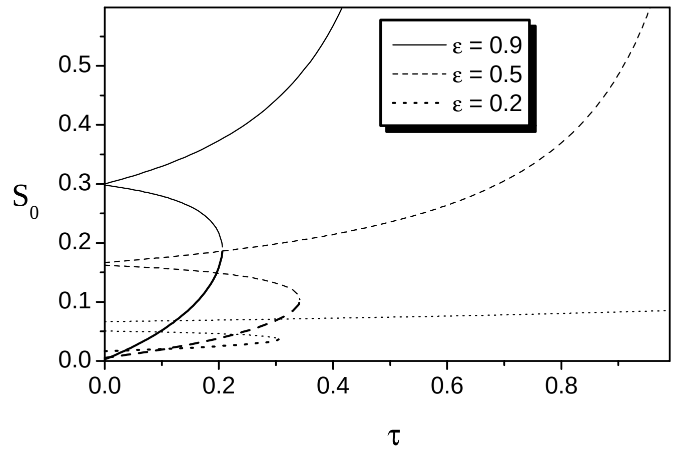


(b)

Fig. 5. Stationary states of the system at $a = 0.2$: (a) order parameter *vs.* control parameter ε; (b) order parameter *vs.* noise correlation time.



(a)

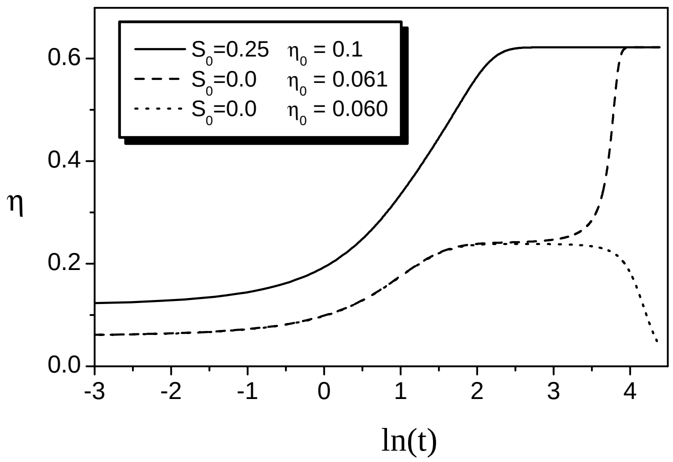


(b)

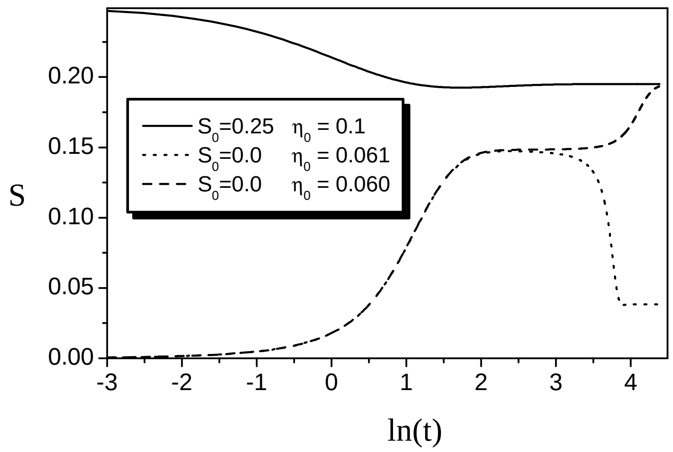
Fig. 6. Stationary states of the system at $a = 0.2$: (a) autocorrelator *vs.* control parameter ε (b) autocorrelator *vs.* noise correlation time.

Let us discuss now the time dependencies corresponding to the phase trajectories. Time dependencies are shown in Figure 7. According to the phase portraits shown in Figure 3, at $\varepsilon \neq [\varepsilon_0^{(1)}, \varepsilon_0^{(2)}]$ the order parameter η falls down monotonically to the point C_0, whereas the autocorrelator can vary nonmonotonically. Inside the domain $[\varepsilon_0^{(1)}, \varepsilon_0^{(2)}]$, where bifurcation occurs, we can see two domains on the phase plane. These domains correspond to the small and large values of η. At small initial values of the order parameter we get the above mentioned behaviour. At intermediate and large magnitudes the first statistical moment reaches the attractive point C. In the vicinity of the saddle point S we get a critical slowing-down and a metastable state can exist for a short time. The similar behaviour can be seen in the vicinity of the separatrix C_0SC.

Let us examine the time dependencies of main averages analytically. We investigate how the phase trajectory attains the point C_0 in the large time limit. Because equations (22, 23) contain power–law dependencies it is



(a)



(b)

Fig. 7. Time dependencies corresponding to the different trajectories on the phase portrait in Figure 3: (a) η *vs.* $\ln(t)$ at $a = 0.2$, $\varepsilon = 0.5$, $\tau = 0.2$; (b) S *vs.* $\ln(t)$ at $a = 0.2$, $\varepsilon = 0.5$, $\tau = 0.2$.

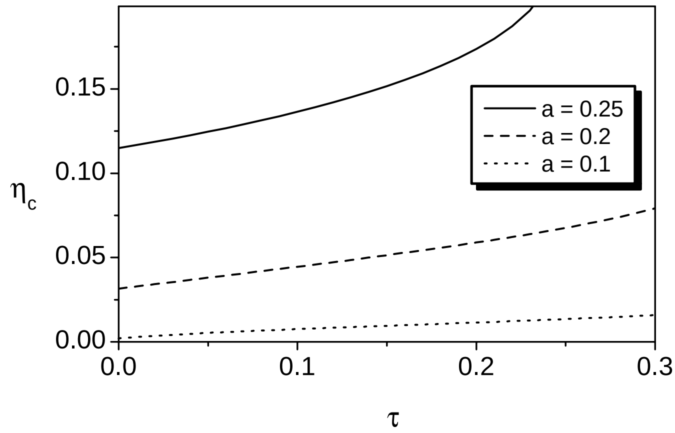


Fig. 8. Critical value of the order parameter η_c *vs.* noise correlation time τ at $\varepsilon = 0.5$ and different values of the noise exponent a.

inconvenient to use the method of ordinary Lyapunov exponent. It will be more useful to apply the generalized Tsallis exponent [11]:

$$e^{qt} \to \exp_q(t) \equiv [1 + (1-q)t]^{1/1-q}. \quad (28)$$

Here q is the generalized index playing the role of the Lyapunov exponent. According to the derivation rule

$$\frac{\partial}{\partial t}\exp_q(t) = \left(\exp_q(t)\right)^q \equiv \exp_q^q(t) \quad (29)$$

and asymptotic behaviour

$$\lim_{t\to 0}\exp_q(t) \to 1 + t, \qquad \lim_{t\to\infty}\exp_q(t) \to [(1-q)t]^{1/1-q} \quad (30)$$

let us assume solutions of equations (22, 23) in the form

$$\eta(t) = m\exp_\mu(t), \quad S(t) = S_0 + n\exp_\nu(t). \quad (31)$$

Inserting equation (31) into equation (22) we receive up to the first order of $m, n \ll 1$:

$$(\epsilon + \kappa S_0)(\epsilon + 4\kappa S_0) = -\kappa\epsilon\alpha_1 m^{p_1}\exp_\mu^{p_1+1-\mu}(t), \quad (32)$$

here we take account of the singular contribution only. In the long–time limit, the function $\exp_\mu^{p_1+1-\mu}(t)$ can be taken to be equal to 1. It yields following definitions for the Lyapunov multiplier m and exponent μ:

$$m = |(\epsilon + \kappa S_0)(\epsilon + 4\kappa S_0)/\kappa\epsilon\alpha_1|^{1/p_1}, \qquad \mu = 1 + \frac{1}{2(1-a)}. \quad (33)$$

Considering the behaviour of the autocorrelator $S(t)$ up to the first order of amplitudes m and n we obtain

$$(\epsilon/2 + 2\kappa S_0) = \exp_\nu^{1-\nu}\left(\varepsilon + \alpha_1 n^{-1}m^{p_1}\exp_\mu^{p_1}(t)\exp_\nu^{-1}(t)\right). \quad (34)$$

In the short-time limit we assume $\exp_\nu^{1-\nu} \to 1$. The long-time asymptote yields $\exp_\mu^{p_1}(t)\exp_\nu^{-1}(t) = \text{const.} \equiv p_1^{-1}$ and, hence, for the exponent and multiplier we have

$$\nu = 2, \qquad n = \frac{1}{\kappa\epsilon p_1}\frac{(\epsilon + \kappa S_0)(\epsilon + 4\kappa S_0)}{\epsilon/2 - \varepsilon + 2\kappa S_0}. \quad (35)$$

According to the obtained time dependencies, we see that the first statistical moment behaves itself in a power–law form, *i.e.* $\eta(t) \propto t^{-2(1-a)}$, and the autocorrelator $S(t)$ follows the hyperbolical dependence $S(t) \propto t^{-1}$.

If we pass through the critical value $\varepsilon_0^{(1)}$ then the system can be ordered and we have to take account of an initial magnitude $\eta(0)$ of the order parameter. Picking $\eta(0)$ larger then a critical value η_c (shown in Fig. 8) we make the system pass to the ordered state. Let us examine how phase trajectories attain the point C. In this case we can not pick up the solution in the form of generalized exponent (28). The latter is applicable for non–linearity effects which are sufficient to fix the amplitudes m and n. In the case under consideration the linear conditions are satisfied hence, we have to use the Mellin transformations

$$\eta(t) = \eta_0 + \int m_q t^q \mathrm{d}q,$$
$$S(t) = S_0 + \int n_q t^q \mathrm{d}q. \tag{36}$$

The evolution equations can be transformed to the system of linear algebraical equations

$$A_{11} m_q + A_{12} n_q = 0,$$
$$A_{21} m_q + A_{22} n_q = 0, \tag{37}$$

where multipliers A_{ij} are functions of the system parameters and coordinates of the point C. The diagonal elements of the matrix $\mathbf{A}$ incorporate terms q/t, so we can rewrite $A_{ii} = A_{ii}^{(0)} + q/t$. System (37) has a solution if $\det|\mathbf{A}| = 0$. So, if we use the following notation $c = -q/t$ then we get

$$\eta(t) = \eta_0 + m \exp(-ct \ln(t)),$$
$$S(t) = S_0 + n \exp(-ct \ln(t)), \tag{38}$$

here amplitudes $m,\ n$ correspond to the index $q = -ct$; c is the real number whose magnitude can be expressed from the condition $\det|\mathbf{A}| = 0$:

$$c = \frac{1}{2}(A_{11}^{(0)} + A_{22}^{(0)}) \left(1 \pm \sqrt{1 - \frac{4(A_{11}^{(0)} A_{22}^{(0)} - A_{12}A_{21})}{(A_{11}^{(0)} + A_{22}^{(0)})^2}}\right). \tag{39}$$

5 Summary

We have considered the effects of self-correlation in multiplicative noise and discuss the dynamics of noise–induced phase transitions in terms of statistical moments. Colouring the multiplicative noise appears to lead to a reentrant phase transition that becomes apparent when the control parameter is increased. Because in the case of white noise there is only a one usual transition along the axis of the control parameter [7], one can conclude that the reentrant transition is a consequence of collaboration between the nonlinearity and colouring the multiplicative noise. We show that colouring does not change the time asymptotes in the vicinity of the attractors related to the disordered and ordered domains. However, it changes the amplitudes of the time dependencies. In our opinion, such behaviour is inherent in all systems with a self-affine phase space.

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